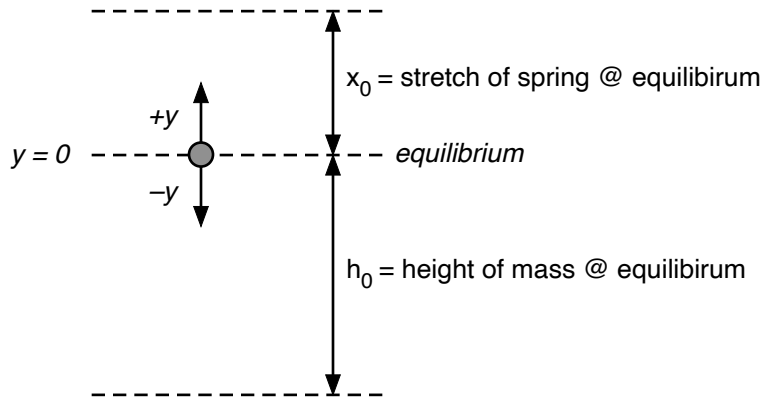
Vertical Springs & Energy

When talking about the energy of an oscillating spring, it turns out that we can ignore gravity and the initial amount of stretch in a spring that is hung vertically. To see that, we will imagine a mass on a spring that is hanging vertically at rest and not moving. In the picture below, the mass is shown, but not the spring.



To hopefully help make it clear: x_0 is the amount that the spring is stretched while the mass is hanging at rest. In this situation, the net force on the mass would be zero, so we can say

$$kx_0 = mg$$

(There have been homework problems based on the above statement.) Now let's start this baby oscillating! If we displace the mass a small distance above or below the equilibrium position, it will begin to oscillate. Let's define the equilibrium position to be y = 0, and positive displacements to be above the equilibrium and negative y to be below the equilibrium.

Now, we can write the following for the energy of the oscillating mass, taking into account movement, the real stretch of the spring and gravity:

$$E = \frac{1}{2}mv^2 + \frac{1}{2}k(x_0 - y)^2 + mg(h_0 + y)$$

Expanding and rewriting the above yields:

$$E = \frac{1}{2}mv^2 + \frac{1}{2}kx_0^2 + \frac{1}{2}ky^2 - kx_0y + mgh_0 + mgy$$

$$E = \frac{1}{2}mv^2 + \frac{1}{2}ky^2 + [\frac{1}{2}kx^2 + mgh_0 - kx_0y + mgy]$$

Since $kx_0 = mg$, this reduces to

$$E = \frac{1}{2}mv^2 + \frac{1}{2}ky^2 + \{\frac{1}{2}kx_0^2 + mgh_0\}$$

Notice there are two dynamic terms, and two static terms. The first two terms in the above are the kinetic energy and the potential energy as if the spring were not stretched or compressed at equilibrium. The second two terms (in the brackets) are constants: the potential energy in the spring at equilibrium and the gravitational potential energy at equilibrium. One can think of those constant terms as the energy it takes to make the system - but it doesn't really impact the oscillations themselves. So, when dealing with a vertical spring, we ignore those terms. The energy of an oscillating system is taken as the kinetic and potential as measured from the equilibrium.